

Radiative Transfer in Relativistic Accretion-Disk Winds

Jun FUKUE

Astronomical Institute, Osaka Kyoiku University, Asahigaoka, Kashiwara, Osaka 582-8582
fukue@cc.osaka-kyoiku.ac.jp

(Received 0 0; accepted 0 0)

Abstract

Radiative transfer in a relativistic accretion disk wind is examined under the plane-parallel approximation in the fully special relativistic treatment. For an equilibrium flow, where the flow speed and the source function are constant, the emergent intensity is analytically obtained. In such an equilibrium flow the usual limb-darkening effect does not appear, since the source function is constant. Due to the Doppler and aberration effects associated with the relativistic motion of winds, however, the emergent intensity is strongly enhanced toward the flow direction. This is the *relativistic peaking effect*. We thus carefully treat and estimate the appearance of relativistic winds and jets, when we observe them in an arbitrary direction.

Key words: accretion, accretion disks — astrophysical jets — gamma-ray bursts — radiative transfer — relativity

1. Introduction

Accretion disks are now widely believed to be energy sources in various active phenomena in the universe (see Kato et al. 2007 for a review). Relating to energetic accretion disks, accretion disk winds have been extensively examined in connection with astrophysical jets and outflows: in bipolar outflows from young stellar objects (YSOs), in outflows from cataclysmic variables (CVs) and supersoft X-ray sources (SSXSs), in relativistic jets from microquasars (μ QSOs), active galactic nuclei (AGNs), quasars (QSOs), and in gamma-ray bursts (GRBs). In particular, intense radiation fields of luminous supercritical accretion disks may be responsible for relativistic jets from super-Eddington sources, such as luminous μ QSOs, GRS 1915+105 and SS 433, luminous QSOs, 3C 273, and energetic GRBs (see, e.g., Fukue 2004 for references). Furthermore, energetic emissions from relativistic jets have been examined, relating to, e.g., gamma-ray blazars and gamma-ray bursts (Dermer, Schlickeiser 1993, 2002; Dermer 1998; Böttcher, Dermer 2002; Dermer et al. 2007).

In such circumstances, radiative transfer in accretion disk winds as well as accretion disks becomes more and more important. Radiative transfer in the standard disk has been investigated in relation to the structure of a static disk atmosphere and the spectral energy distribution from the disk surface (e.g., Meyer, Meyer-Hofmeister 1982; Cannizzo, Wheeler 1984). Furthermore, gray and non-gray models of accretion disks were constructed under numerical treatments (Křiž, Hubeny 1986; Shaviv, Wehrse 1986; Adam et al. 1988; Mineshige, Wood 1990; Ross et al. 1992; Shimura, Takahara 1993; Hubeny, Hubeny 1997, 1998; Hubeny et al. 2000, 2001; Davis et al. 2005; Hui et al. 2005) and under analytical ones (Hubeny 1990; Artemova et al. 1996; Fukue, Akizuki 2006a).

Radiative transfer in the accretion disk wind, on the other hand, has not been well considered both in the non-relativistic and relativistic regimes. For example, transformation properties of disk radiation fields in the proper frame of a relativistic jet were examined by, e.g., Dermer and Schlickeiser (2002). In these earlier works, however, the radiation fields are set to be external sources, and the radiation transfer was not considered. Recently, radiative transfer in a moving disk atmosphere was firstly investigated in the subrelativistic regime (Fukue 2005a, 2006a), and in the relativistic regime (Fukue 2005b, 2006b; Fukue, Akizuki 2006b). In these studies, however, only the radiative moments were obtained under the moment formalism, and the specific intensity was not solved. Hence, in Fukue (2007) the specific intensity from an accretion disk wind was obtained in the subrelativistic regime, where the flow speed v is of the order of $(v/c)^1$.

In this paper, we thus extend the previous work. Namely, we examine radiative transfer in the relativistic accretion disk wind, which is assumed to blow off from the luminous disk in the vertical direction (plane-parallel approximation), with the relativistic speed up to the order of c .

In the next section we describe the basic equations. In section 3, we show analytical solutions of the specific intensity. The final section is devoted to concluding remarks.

2. Relativistic Radiative Transfer Equation

Let us suppose a luminous flat disk, deep inside which gravitational or nuclear energy is released via viscous heating or other processes. The radiation energy is transported in the vertical direction, and the disk gas, itself, also moves in the vertical direction as a *disk wind* due to the action of radiation pressure (i.e., plane-parallel approximation). For simplicity, in the present paper, the radiation field is considered to be sufficiently intense that both the gravitational field of, e.g., the central object and the gas pressure can be ignored. We also assume the gray approximation, where the opacities do not depend on the frequency. As for the order of the flow velocity v , we consider the fully special relativistic regime.

The radiative transfer equations are given in several literatures (Chandrasekhar 1960; Mihalas 1970; Rybicki, Lightman 1979; Mihalas, Mihalas 1984; Shu 1991; Kato et al. 1998, 2007; Peraiah 2002; Castor 2004). The radiative transfer equation in the fully relativistic form is given in, e.g., the appendix E of Kato et al. (2007) in general and vertical forms.

In a general form the radiative transfer equation in the inertial (fixed) frame is expressed as

$$\begin{aligned} \frac{1}{c} \frac{\partial I}{\partial t} + (\mathbf{l} \cdot \nabla) I = \rho \gamma^{-3} \left(1 - \frac{\mathbf{v} \cdot \mathbf{l}}{c} \right)^{-3} & \left[\frac{j_0}{4\pi} - (\kappa_0^{\text{abs}} + \kappa_0^{\text{sca}}) \gamma^4 \left(1 - \frac{\mathbf{v} \cdot \mathbf{l}}{c} \right)^4 I \right. \\ & + \frac{\kappa_0^{\text{sca}}}{4\pi} \frac{3}{4} \gamma^{-2} \left(1 - \frac{\mathbf{v} \cdot \mathbf{l}}{c} \right)^{-2} \left\{ \gamma^4 \left[\left(1 - \frac{\mathbf{v} \cdot \mathbf{l}}{c} \right)^2 + \left(\frac{v^2}{c^2} - \frac{\mathbf{v} \cdot \mathbf{l}}{c} \right)^2 \right] cE \right. \\ & + 2\gamma^2 \left(\frac{v^2}{c^2} - \frac{\mathbf{v} \cdot \mathbf{l}}{c} \right) \mathbf{F} \cdot \mathbf{l} - 2\gamma^4 \left[\left(1 - \frac{\mathbf{v} \cdot \mathbf{l}}{c} \right)^2 + \left(1 - \frac{\mathbf{v} \cdot \mathbf{l}}{c} \right) \left(\frac{v^2}{c^2} - \frac{\mathbf{v} \cdot \mathbf{l}}{c} \right) \right] \frac{\mathbf{v} \cdot \mathbf{F}}{c} \\ & \left. \left. + l_i l_j c P^{ij} - 2\gamma^2 \left(1 - \frac{\mathbf{v} \cdot \mathbf{l}}{c} \right) v_i l_j P^{ij} + 2\gamma^4 \left(1 - \frac{\mathbf{v} \cdot \mathbf{l}}{c} \right)^2 \frac{v_i v_j P^{ij}}{c} \right] \right\}, \end{aligned} \quad (1)$$

Here, \mathbf{v} is the flow velocity, c is the speed of light, and $\gamma (= 1/\sqrt{1-v^2/c^2})$ is the Lorentz factor. In the left-hand side the frequency-integrated specific intensity I and the direction cosine \mathbf{l} are quantities measured in the inertial (fixed) frame. In the right-hand side, the mass density ρ , the frequency-integrated mass emissivity j_0 , the frequency-integrated mass absorption coefficient κ_0^{abs} , and the frequency-integrated mass scattering coefficient κ_0^{sca} are quantities measured in the comoving (fluid) frame, whereas the frequency-integrated radiation energy density E , the frequency-integrated radiative flux \mathbf{F} , and the frequency-integrated radiation stress tensor P^{ij} are quantities measured in the inertial (fixed) frame.

In the plane-parallel geometry with the vertical axis z and the direction cosine $\mu (= \cos\theta)$, the transfer equation is expressed as

$$\begin{aligned} \mu \frac{dI}{dz} = \rho \frac{1}{\gamma^3 (1 - \beta\mu)^3} & \left[\frac{j_0}{4\pi} - (\kappa_0^{\text{abs}} + \kappa_0^{\text{sca}}) \gamma^4 (1 - \beta\mu)^4 I + \frac{\kappa_0^{\text{sca}}}{4\pi} \frac{3}{4} \gamma^2 \left\{ \left[1 + \frac{(\mu - \beta)^2}{(1 - \beta\mu)^2} \beta^2 + \frac{(1 - \beta^2)^2}{(1 - \beta\mu)^2} \frac{1 - \mu^2}{2} \right] cE \right. \right. \\ & \left. \left. - \left[1 + \frac{(\mu - \beta)^2}{(1 - \beta\mu)^2} \right] 2F\beta + \left[\beta^2 + \frac{(\mu - \beta)^2}{(1 - \beta\mu)^2} - \frac{(1 - \beta^2)^2}{(1 - \beta\mu)^2} \frac{1 - \mu^2}{2} \right] cP \right\} \right], \end{aligned} \quad (2)$$

where $\beta (= v/c)$ is the normalized vertical speed, and F and P are the vertical component of the radiative flux and the radiation stress tensor measured in the inertial frame, respectively.

For the convenience of readers, we shall show the full set of radiation hydrodynamical equations under the plane-parallel approximation, although we do not use all of them in this paper.

For matter, the continuity equation, the equation of motion, and the energy equation become, respectively,

$$\rho c u = \rho \gamma \beta c = J \quad (= \text{const.}), \quad (3)$$

$$c^2 u \frac{du}{dz} = c^2 \gamma^4 \beta \frac{d\beta}{dz} = -\frac{d\psi}{dz} - \gamma^2 \frac{c^2}{\varepsilon + p} \frac{dp}{dz} + \frac{\rho c^2}{\varepsilon + p} \frac{\kappa_0^{\text{abs}} + \kappa_0^{\text{sca}}}{c} \gamma^3 \left[F(1 + \beta^2) - (cE + cP)\beta \right], \quad (4)$$

$$0 = \frac{q^+}{\rho} - \left(j_0 - \kappa_0^{\text{abs}} c E \gamma^2 - \kappa_0^{\text{abs}} c P u^2 + 2\kappa_0^{\text{abs}} F \gamma u \right), \quad (5)$$

where $u (= \gamma\beta)$ is the vertical four velocity, J the mass-loss rate per unit area, ψ the gravitational potential, ε the internal energy per unit proper volume, p the gas pressure, and q^+ the internal heating. In the energy equation (5) the advection terms in the left-hand side are dropped under the present cold approximation.

For radiation, the zeroth and first moment equations, and the closure relation become, respectively,

$$\frac{dF}{dz} = \rho \gamma \left[j_0 - \kappa_0^{\text{abs}} c E + \kappa_0^{\text{sca}} (cE + cP) \gamma^2 \beta^2 + \kappa_0^{\text{abs}} F \beta - \kappa_0^{\text{sca}} F (1 + \beta^2) \gamma^2 \beta \right]. \quad (6)$$

$$\frac{dP}{dz} = \frac{\rho\gamma}{c} \left[j_0\beta - \kappa_0^{\text{abs}} F + \kappa_0^{\text{abs}} cP\beta - \kappa_0^{\text{sca}} F\gamma^2(1 + \beta^2) + \kappa_0^{\text{sca}} (cE + cP)\gamma^2\beta \right], \quad (7)$$

$$cP(1 - f\beta^2) = cE(f - \beta^2) + 2F\beta(1 - f), \quad (8)$$

where $f(\tau, \beta)$ is the variable Eddington factor, which is defined by $f = P_{\text{co}}/E_{\text{co}}$, E_{co} and P_{co} being the comoving quantities, and generally depends on the velocity and its gradient as well as the optical depth (Fukue 2006b, 2007b).

Eliminating j_0 using the energy equation (5), the transfer equation (2) becomes

$$\begin{aligned} \mu \frac{dI}{dz} = \rho \frac{1}{\gamma^3(1 - \beta\mu)^3} & \left[-(\kappa_0^{\text{abs}} + \kappa_0^{\text{sca}}) \gamma^4 (1 - \beta\mu)^4 I + \frac{q^+}{4\pi\rho} + \frac{\kappa_0^{\text{abs}}}{4\pi} \gamma^2 (cE - 2F\beta + \beta^2 cP) \right. \\ & + \frac{\kappa_0^{\text{sca}}}{4\pi} \frac{3}{4} \gamma^2 \left\{ \left[1 + \frac{(\mu - \beta)^2}{(1 - \beta\mu)^2} \beta^2 + \frac{(1 - \beta^2)^2}{(1 - \beta\mu)^2} \frac{1 - \mu^2}{2} \right] cE \right. \\ & \left. \left. - \left[1 + \frac{(\mu - \beta)^2}{(1 - \beta\mu)^2} \right] 2F\beta + \left[\beta^2 + \frac{(\mu - \beta)^2}{(1 - \beta\mu)^2} - \frac{(1 - \beta^2)^2}{(1 - \beta\mu)^2} \frac{1 - \mu^2}{2} \right] cP \right\} \right]. \quad (9) \end{aligned}$$

Introducing the optical depth defined by

$$d\tau = -(\kappa_0^{\text{abs}} + \kappa_0^{\text{sca}}) \rho dz, \quad (10)$$

the transfer equation (9) finally becomes

$$\begin{aligned} \mu \frac{dI}{d\tau} = \frac{1}{\gamma^3(1 - \beta\mu)^3} & \left[\gamma^4 (1 - \beta\mu)^4 I - \frac{q^+}{4\pi(\kappa_0^{\text{abs}} + \kappa_0^{\text{sca}})\rho} - \frac{1}{4\pi} \frac{\kappa_0^{\text{abs}}}{\kappa_0^{\text{abs}} + \kappa_0^{\text{sca}}} \gamma^2 (cE - 2F\beta + \beta^2 cP) \right. \\ & - \frac{1}{4\pi} \frac{\kappa_0^{\text{sca}}}{\kappa_0^{\text{abs}} + \kappa_0^{\text{sca}}} \frac{3}{4} \gamma^2 \left\{ \left[1 + \frac{(\mu - \beta)^2}{(1 - \beta\mu)^2} \beta^2 + \frac{(1 - \beta^2)^2}{(1 - \beta\mu)^2} \frac{1 - \mu^2}{2} \right] cE \right. \\ & \left. \left. - \left[1 + \frac{(\mu - \beta)^2}{(1 - \beta\mu)^2} \right] 2F\beta + \left[\beta^2 + \frac{(\mu - \beta)^2}{(1 - \beta\mu)^2} - \frac{(1 - \beta^2)^2}{(1 - \beta\mu)^2} \frac{1 - \mu^2}{2} \right] cP \right\} \right]. \quad (11) \end{aligned}$$

Similarly, radiation hydrodynamical equations (4), (6), and (7), with the help of continuity equation (3) and the closure relation (8), become (cf. Fukue 2005b, 2006b)

$$c^2 J \gamma^3 \frac{d\beta}{d\tau} = -\gamma^3 [F(1 + \beta^2) - (cE + cP)\beta] = -\gamma \frac{F(f + \beta^2) - cP(1 + f)\beta}{f - \beta^2}, \quad (12)$$

$$\frac{dF}{d\tau} = -\frac{q^+ \gamma}{(\kappa_0^{\text{abs}} + \kappa_0^{\text{sca}})\rho} + \gamma^3 \beta [F(1 + \beta^2) - (cE + cP)\beta] = -\frac{q^+ \gamma}{(\kappa_0^{\text{abs}} + \kappa_0^{\text{sca}})\rho} + \gamma \beta \frac{F(f + \beta^2) - cP(1 + f)\beta}{f - \beta^2}, \quad (13)$$

$$\frac{dP}{d\tau} = -\frac{q^+ u}{(\kappa_0^{\text{abs}} + \kappa_0^{\text{sca}})\rho} + \gamma^3 [F(1 + \beta^2) - (cE + cP)\beta] = -\frac{q^+ u}{(\kappa_0^{\text{abs}} + \kappa_0^{\text{sca}})\rho} + \gamma \frac{F(f + \beta^2) - cP(1 + f)\beta}{f - \beta^2}. \quad (14)$$

Here, we dropped the gravitational and pressure forces. For such a radiation-dominated flow, where the gravitational and pressure forces are neglected, there are two integrals (Fukue 2005b, 2006b):

$$c^2 J \gamma + F = c^2 J + F_0 - \int_{\tau_0}^{\tau} \frac{q^+}{(\kappa_0^{\text{abs}} + \kappa_0^{\text{sca}})\rho} \gamma d\tau, \quad (15)$$

$$c^2 J u + cP = cP_0 - \int_{\tau_0}^{\tau} \frac{q^+}{(\kappa_0^{\text{abs}} + \kappa_0^{\text{sca}})\rho} u d\tau. \quad (16)$$

3. Analytical Solutions

In order to solve the transfer equation (11) analytically, we suppose several assumptions; we assume that there is no internal heating ($q^+ = 0$) and the flow reaches the equilibrium state, where the flow speed is almost constant ($\beta = \text{const.}$). In this case, from equations (15) and (16), and the closure relation (8), the radiative flux F , the radiation stress tensor P , and the radiation energy density E are all constant. We further assume that the disk has a finite optical depth, and there exists a uniform isotropic source of intensity I_0 at the disk equator of optical depth τ_0 .

Now, it is not so difficult to integrate the transfer equation (11). We first rewrite equation (11) symbolically as

$$\mu \frac{dI}{d\tau} = \gamma(1 - \beta\mu)I - S'_a - S'_s, \quad (17)$$

where

$$S'_a = \frac{S_a}{\gamma^3(1-\beta\mu)^3} = \frac{1}{4\pi} \frac{\kappa_0^{\text{abs}}}{\kappa_0^{\text{abs}} + \kappa_0^{\text{sca}}} \frac{1}{\gamma(1-\beta\mu)^3} (cE - 2F\beta + \beta^2 cP) \quad (18)$$

$$S'_s = \frac{S_s}{\gamma^3(1-\beta\mu)^3} = \frac{1}{4\pi} \frac{\kappa_0^{\text{sca}}}{\kappa_0^{\text{abs}} + \kappa_0^{\text{sca}}} \frac{3}{4} \frac{1}{\gamma(1-\beta\mu)^3} \left\{ \left[1 + \frac{(\mu-\beta)^2}{(1-\beta\mu)^2} \beta^2 + \frac{(1-\beta^2)^2}{(1-\beta\mu)^2} \frac{1-\mu^2}{2} \right] cE \right. \\ \left. - \left[1 + \frac{(\mu-\beta)^2}{(1-\beta\mu)^2} \right] 2F\beta + \left[\beta^2 + \frac{(\mu-\beta)^2}{(1-\beta\mu)^2} - \frac{(1-\beta^2)^2}{(1-\beta\mu)^2} \frac{1-\mu^2}{2} \right] cP \right\}, \quad (19)$$

where $S'_a + S'_s$ is the Doppler boosted source function and $S_a + S_s$ is the non-boosted source function, both being independent of the optical depth under the present approximation.

Under the above situations, we can formally integrate the transfer equation (11), similar to Fukue and Akizuki (2006) and Fukue (2007). After several partial integrations, we obtain both an outward intensity $I(\tau, \mu, \beta)$ ($\mu > 0$) and an inward intensity $I(\tau, -\mu, \beta)$ as

$$I(\tau, \mu, \beta) = \frac{S_a + S_s}{\gamma^4(1-\beta\mu)^4} \left[1 - e^{-\frac{\gamma(1-\beta\mu)}{\mu}(\tau-\tau_0)} \right] + I(\tau_0, \mu) e^{-\frac{\gamma(1-\beta\mu)}{\mu}(\tau-\tau_0)}, \quad (20)$$

$$I(\tau, -\mu, \beta) = \frac{S_a + S_s}{\gamma^4(1-\beta\mu)^4} \left[1 - e^{-\frac{\gamma(1-\beta\mu)}{\mu}\tau} \right], \quad (21)$$

where $I(\tau_0, \mu)$ is the boundary value at the wind base on the luminous disk. These analytical solutions are reduced to those obtained in Fukue (2007) in the subrelativistic limit of $\gamma = 1$.

In general case with finite optical depth τ_0 and uniform incident intensity I_0 from the disk, the boundary value $I(\tau_0, \mu, \beta)$ of the outward intensity I consists of two parts:

$$I(\tau_0, \mu, \beta) = I_0 + I(\tau_0, -\mu, \beta), \quad (22)$$

where I_0 is the uniform incident intensity and $I(\tau_0, -\mu, \beta)$ is the *inward* intensity from the backside of the disk beyond the midplane. Determining $I(\tau_0, -\mu, \beta)$ from equation (21), we finally obtain the outward intensity as

$$I(\tau, \mu, \beta) = \frac{S_a + S_s}{\gamma^4(1-\beta\mu)^4} \left[1 - e^{-\frac{\gamma(1-\beta\mu)}{\mu}(\tau-2\tau_0)} \right] + I_0 e^{-\frac{\gamma(1-\beta\mu)}{\mu}(\tau-\tau_0)}. \quad (23)$$

Finally, the emergent intensity $I(0, \mu, \beta)$ emitted from the wind top becomes

$$I(0, \mu, \beta) = \frac{S_a + S_s}{\gamma^4(1-\beta\mu)^4} \left[1 - e^{-\frac{\gamma(1-\beta\mu)}{\mu}2\tau_0} \right] + I_0 e^{-\frac{\gamma(1-\beta\mu)}{\mu}\tau_0}, \\ \sim \frac{S_a + S_s}{\gamma^4(1-\beta\mu)^4} \quad \text{for large } \tau_0. \quad (24)$$

In order to calculate the source functions, S_a and S_s , we consider two special cases below: a terminal case and an optically thin limit.

3.1. Terminal Case

When the flow speed is almost the equilibrium one and the radiation field is almost constant, then the values of the quantities of radiation fields are almost equal to those at the flow top.

At the flow top of a moving photosphere at a relativistic speed, the usual boundary conditions for a static atmosphere is inadequate, as already pointed out in Fukue (2005b). Namely, the radiation field just above the wind top changes when the gas itself does move upward, since the direction and intensity of radiation change due to the relativistic aberration and Doppler effect (cf. Kato et al. 1998, 2007; Fukue 2000). If a flat infinite plane with surface intensity I_s in the comoving frame is not static, but moving upward at a speed v_s ($= c\beta_s$, and the corresponding Lorentz factor is γ_s), where the subscript s denotes the values at the surface, then, just above the surface, the radiation energy density E_s , the radiative flux F_s , and the radiation pressure P_s measured in the inertial frame become, respectively,

$$cE_s = 2\pi I_s \gamma_s^2 \frac{3 + 3\beta_s + \beta_s^2}{3}, \quad (25)$$

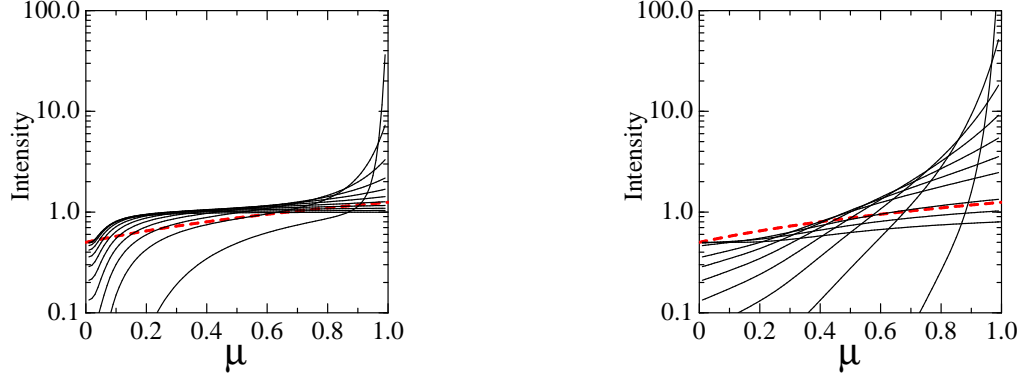


Fig. 1. Normalized emergent intensity as a function of μ for several values of β : (a) $\tau_0 = 0.1$ and (b) $\tau_0 = 1$ in the terminal case. The values of β are 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, and 0.99: as the flow speed becomes large, the intensity at $\mu = 1$ is large. The dashed line is for the usual Milne-Eddington solution for the plane-parallel case.

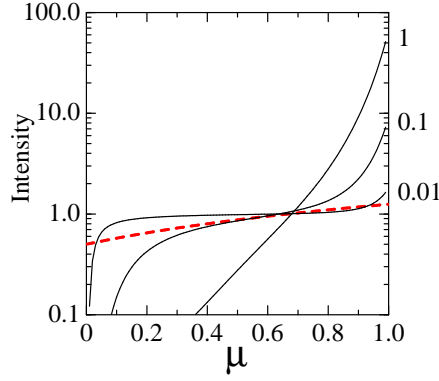


Fig. 2. Normalized emergent intensity as a function of μ for several values of τ_0 in the case of $\beta = 0.9$ in the terminal case. The values of τ_0 are 0.01, 0.1, and 1. The dashed line is for the usual Milne-Eddington solution for the plane-parallel case.

$$F_s = 2\pi I_s \gamma_s^2 \frac{3 + 8\beta_s + 3\beta_s^2}{6}, \quad (26)$$

$$cP_s = 2\pi I_s \gamma_s^2 \frac{1 + 3\beta_s + 3\beta_s^2}{3}. \quad (27)$$

In this case the non-boosted source functions are calculated as

$$S_a = \frac{\kappa_0^{\text{abs}}}{\kappa_0^{\text{abs}} + \kappa_0^{\text{sca}}} \frac{I_s}{2}, \quad S_s = \frac{\kappa_0^{\text{sca}}}{\kappa_0^{\text{abs}} + \kappa_0^{\text{sca}}} \frac{I_s}{2}; \quad S_a + S_s = \frac{I_s}{2}, \quad (28)$$

which depend on neither the flow speed nor direction cosine. Even if the non-boosted source function is constant, the emergent intensity (24) does depend on the flow speed and the direction cosine. Examples of solutions are shown in figures 1 and 2.

In figure 1, the emergent intensity $I(0, \mu, \beta)$ normalized by I_s is shown for several values of β and τ_0 as a function of μ .

In the case of small τ_0 (figure 1a), when the flow speed is small, the normalized emergent intensity is almost unity except for small μ direction, since the uniform source is seen except for small μ direction, where the source function is seen. When the flow speed becomes large, however, the emergent intensity becomes remarkably anisotropic; it decreases in the edgeward direction, whereas it greatly increases in the poleward direction. This is the *relativistic peaking effect*, which originates from the relativistic Doppler effect and aberration.

In the case of $\tau_0 = 1$ (figure 1b), the emergent intensity for small β is reduced since the optical depth becomes large. However, the relativistic peaking effect becomes effective as the disk optical depth becomes large.

In figure 2, the normalized emergent intensity $I(0, \mu, \beta)$ is shown for several values of τ_0 in the case of $\beta = 0.9$. As is seen in figure 2, the relativistic peaking effect strongly depends on the disk optical depth.

3.2. Optically Thin Limit

We next consider the flow at a constant speed using the closure relation (8) in the optically thin limit.

Eliminating E by the closure relation (8), the source functions (18) and (19) are expressed as

$$\begin{aligned} S_a &= \frac{1}{4\pi} \frac{\kappa_0^{\text{abs}}}{\kappa_0^{\text{abs}} + \kappa_0^{\text{sca}}} \frac{(1 + \beta^2)cP - 2F\beta}{f - \beta^2} \\ &= \frac{1 - A}{4\pi} \frac{(1 + \beta^2)cP - 2F\beta}{f - \beta^2}, \end{aligned} \quad (29)$$

$$\begin{aligned} S_s &= \frac{1}{4\pi} \frac{\kappa_0^{\text{sca}}}{\kappa_0^{\text{abs}} + \kappa_0^{\text{sca}}} \frac{(1 + \beta^2)cP - 2F\beta}{f - \beta^2} \frac{3}{4} \left[1 + \frac{(\mu - \beta)^2}{(1 - \beta\mu)^2} f + \frac{(1 - \beta^2)(1 - \mu^2)}{(1 - \beta\mu)^2} \frac{1 - f}{2} \right] \\ &= \frac{A}{4\pi} \frac{(1 + \beta^2)cP - 2F\beta}{f - \beta^2} \frac{3}{4} \left[1 + \frac{(\mu - \beta)^2}{(1 - \beta\mu)^2} f + \frac{(1 - \beta^2)(1 - \mu^2)}{(1 - \beta\mu)^2} \frac{1 - f}{2} \right], \end{aligned} \quad (30)$$

where

$$A \equiv \frac{\kappa_0^{\text{sca}}}{\kappa_0^{\text{abs}} + \kappa_0^{\text{sca}}} \quad (31)$$

is the scattering albedo.

In the optically thin limit, we can easily obtain the quantities of radiation fields and the *velocity-dependent* Eddington factor (cf. Fukue 2006b, 2007b; Koizumi and Umemura 2007). That is to say, if there exists a uniform source of intensity I_0 at the disk equator, the radiation energy density E , the radiative flux F , and the radiation stress tensor P in the inertial (fixed) frame are respectively calculated as

$$cE = 2\pi I_0, \quad (32)$$

$$F = \pi I_0, \quad (33)$$

$$cP = \frac{2}{3}\pi I_0. \quad (34)$$

Hence, from the closure relation (8) the velocity-dependent variable Eddington factor $f(\beta)$ is derived as

$$f(\beta) = \frac{1 - 3\beta + 3\beta^2}{3 - 3\beta + \beta^2}. \quad (35)$$

Using these expressions, the source functions S_a and S_s are explicitly expressed as a function of β and μ :

$$S_a = \frac{1 - A}{2} \frac{I_0}{3} \frac{3 - 3\beta + \beta^2}{1 - \beta^2}, \quad (36)$$

$$S_s = \frac{A}{2} \frac{I_0}{4} \frac{(1 - \beta\mu)^2(3 - 3\beta + \beta^2) + (\mu - \beta)^2(1 - 3\beta + \beta^2) + (1 - \beta^2)^2(1 - \mu^2)}{(1 - \beta^2)(1 - \beta\mu)^2}. \quad (37)$$

The emergent intensity (24) does also depend both on the flow speed and the direction cosine. Examples of solutions are shown in figures 3 and 4.

In figure 3, the emergent intensity $I(0, \mu, \beta)$ normalized by I_0 is shown for several values of β and τ_0 as a function of μ .

The qualitative behavior of the emergent intensity is similar to that of the terminal case. That is, in the case of small τ_0 (figure 3a), when the flow speed is small, the normalized emergent intensity is almost unity except for small μ direction, since the uniform source is seen except for small μ direction, where the source function is seen. When the flow speed becomes large, however, the *relativistic peaking effect* becomes prominent. In the case of $\tau_0 = 1$ (figure 3b), the emergent intensity for small β is reduced since the optical depth becomes large. However, the relativistic peaking effect becomes effective as the disk optical depth becomes large.

In figure 4, the emergent intensity and the source functions normalized by I_0 are shown for several values of β . In figure 4, thin solid curves represent S_s/A , $S_a + S_s$ ($A = 0.5$), and $S_a/(1 - A)$, from bottom to top, respectively, whereas thick solid curves denote $I(0, \mu, \beta)$ ($A = 0.5$).

As is seen in figure 4 and equation (36), the source function S_a does depend only on the flow speed. This is just because, in the present definition, the source function S_a is proportional to the radiation energy density E_{co} in the comoving frame, since $cE_{\text{co}} = cE - 2F\beta + \beta^2 cP$. On the other hand, the source function S_s depends on the direction cosine as well as the flow speed. It is small both in the forward and backward directions, while it becomes the maximum in the direction at $\mu = \beta$.

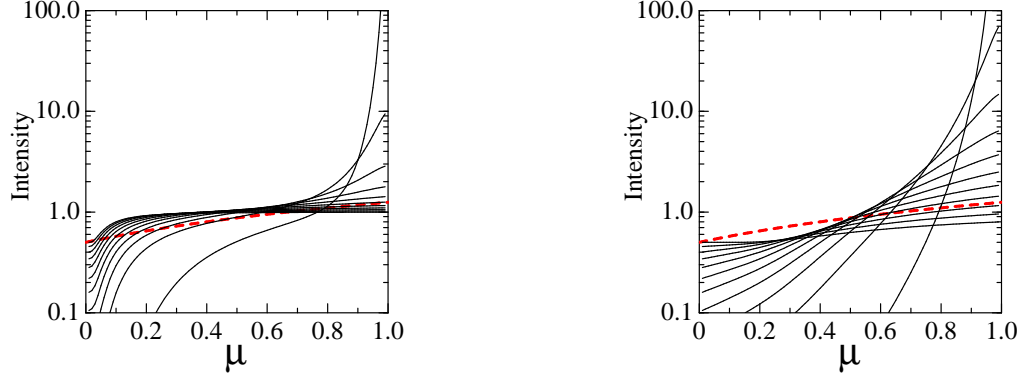


Fig. 3. Normalized emergent intensity as a function of μ for several values of β : (a) $\tau_0 = 0.1$ and (b) $\tau_0 = 1$ in the optically thin limit. The values of β are 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, and 0.99: as the flow speed becomes large, the intensity at $\mu = 1$ is large. The dashed line is for the usual Milne-Eddington solution for the plane-parallel case.

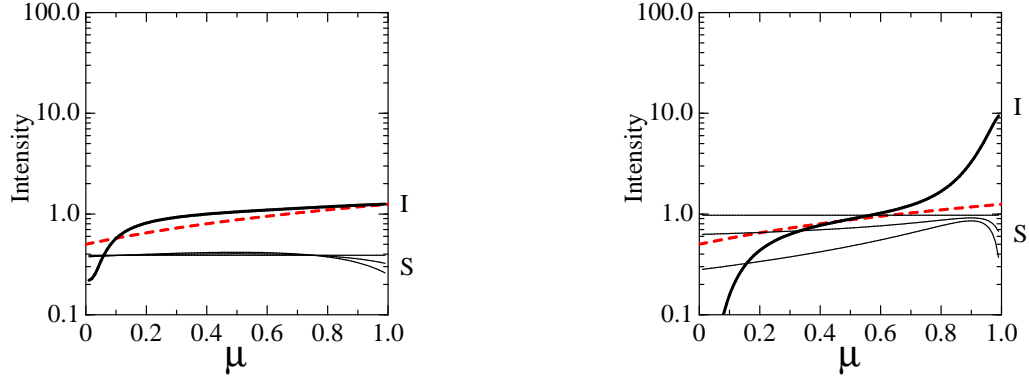


Fig. 4. Normalized emergent intensity and source functions as a function of μ for several values of β in the case of $\tau_0 = 0.1$ in the optically thin limit. The values of β are (a) 0.5 and (b) 0.9. Thin solid curves represent S_s/A , $S_a + S_s$ ($A = 0.5$), and $S_a/(1 - A)$, from bottom to top, respectively, whereas thick solid curves denote $I(0, \mu, \beta)$ ($A = 0.5$). The dashed line is for the usual Milne-Eddington solution for the plane-parallel case.

4. Concluding Remarks

In this paper we have examined the radiative transfer problem in an accretion disk wind under the plane-parallel approximation in the fully special relativistic regime. For an equilibrium flow, where the radiative quantities and source function are constant, we analytically obtain the specific and emergent intensities. We found that the emergent intensity depends on the flow speed as well as the direction cosine, and exhibits a relativistic peaking effect. As a result, a wind luminosity would be overestimated by a pole-on observer and underestimated by an edge-on observer, when we observe an accretion disk wind (cf. Sumitomo et al. 2007; Nishiyama et al. 2007).

It should be noted that the *apparent* optical depth in the relativistically moving media. Abramowicz et al. (1991) pointed out that the optical depth in the relativistic flow decreases as $\gamma(1 - \beta\mu)\tau$ toward the downstream direction, due to the Doppler and aberration effects. Inspecting equation (11) or solution (23), we find that in the present case the optical depth τ is apparently replaced by $\gamma(1 - \beta\mu)\tau$. This is just consistent with the results by Abramowicz et al. (1991).

In this paper we only examined the frequency-integrated intensity under the plane-parallel and gray approximations. It should be briefly remarked on the frequency dependence; i.e., the frequency-dependent intensity I_ν . As long as the opacity is gray, the spectral transformation is determined by the relativistic effect in the present situation. Namely, for the frequency-integrated intensity the relativistic invariant is I/ν^4 , and this effect appears in solution (23) and other equations as a factor of $[\gamma(1 - \beta\mu)]^4$. Since the relativistic invariant for the frequency-dependent intensity is I_ν/ν^3 , the corresponding factor should be changed as $[\gamma(1 - \beta\mu)]^3$. On the other hand, the relativistic modification

in the optical depth discussed above is not changed in the frequency-dependent case. Hence, the present results may be valid for the frequency-dependent emergent intensity; e.g., the incident intensity $I_{\nu 0}$ would be boosted in the polar direction, according to solution (23), but $[\gamma(1 - \beta\mu)]^4$ is replaced by $[\gamma(1 - \beta\mu)]^3$.

The present study can be applied to energetic jets in, e.g., gamma-ray blazars and gamma-ray bursts. The effect of the relativistic jets on the emergent spectrum has been studied in several literatures (Dermer, Schlickeiser 1993, 2002; Dermer 1998; Böttcher, Dermer 2002; Dermer et al. 2007). In these earlier works, however, the radiation transfer was not treated at all. Hence, the present approach may be very useful in these fields of active phenomena.

The radiative transfer problem investigated in the present paper must be quite *fundamental problems* for accretion disk physics and astrophysical jet formation. In order to demonstrate the existence of the relativistic peaking effect, we have imposed various assumptions, including a constant flow speed, gray approximation, no heating source, and so on. By relaxing these assumptions and integrating the relativistic transfer equation numerically, we could obtain the emergent intensity and spectra more quantitatively. These are left as future works.

The author would like to thank S. Kato, S. Mineshige, M. Umemura, T. Koizumi, and C. Akizuki for enlightening and stimulating discussions. This work has been supported in part by a Grant-in-Aid for Scientific Research (18540240 J.F.) of the Ministry of Education, Culture, Sports, Science and Technology.

References

- Abramowicz, M. A., Novilcov, I. D., & Paczyński B. 1991, ApJ, 369, 175
- Adam, J., Störzer, H., Shaviv, G., & Wehrse, R. 1988, A&A, 193, L1
- Akizuki, C., & Fukue, J. 2007, PASJ, submitted
- Artemova, I. V., Bisnovatyi-Kogan, G. S., Björnsson, G., & Novikov, I. D. 1996, ApJ, 456, 119
- Böttcher, M., Dermer, C.D. 2002, ApJ, 564, 86
- Castor, J.I. 2004, Radiation Hydrodynamics (Cambridge: Cambridge University Press)
- Chandrasekhar, S. 1960, Radiative Transfer (New York: Dover Publishing, Inc.)
- Cannizzo, J. K., & Wheeler, J. C. 1984, ApJS, 55, 367
- Davis, S. W., Blaes, O. M., Hubeny, I., & Turner, N. J. 2005, ApJ, 621, 372
- Dermer, C.D. 1998, ApJ, 501, L157
- Dermer, C.D., & Schlickeiser, R. 1993, ApJ, 416, 458
- Dermer, C.D., & Schlickeiser, R. 2002, ApJ, 575, 667
- Dermer, C.D., Ramirez-Ruiz, E., & Truong, L. 2007, ApJ, 664, L67
- Fukue, J. 2000, PASJ, 52, 829
- Fukue, J. 2004, PASJ, 56, 181
- Fukue, J. 2005a, PASJ, 57, 841
- Fukue, J. 2005b, PASJ, 57, 1023
- Fukue, J. 2006a, PASJ, 58, 187
- Fukue, J. 2006b, PASJ, 58, 461
- Fukue, J. 2007, PASJ, 59, in press
- Fukue, J., & Akizuki, C. 2006a, PASJ, 58, 1039
- Fukue, J., & Akizuki, C. 2006b, PASJ, 58, 1073
- Hubeny, I. 1990, ApJ, 351, 632
- Hubeny, I., & Hubeny, V. 1997, ApJ, 484, L37
- Hubeny, I., & Hubeny, V. 1998, ApJ, 505, 558
- Hubeny, I., Agol, E., Blaes, O., & Krolik, J. H. 2000, ApJ, 533, 710
- Hubeny, I., Blaes, O., Krolik, J. H., & Agol, E. 2001, ApJ, 559, 680
- Hui, Y., Krolik, J. H. & Hubeny, I. 2005, 625, 913
- Kato, S., Fukue, J., & Mineshige, S. 1998, Black-Hole Accretion Disks (Kyoto: Kyoto University Press)
- Kato, S., Fukue, J., & Mineshige, S. 2007, Black-Hole Accretion Disks – Towards a New Paradigm – (Kyoto: Kyoto University Press)
- Koizumi, T., & Umemura, M. 2007, submitted to Křiz, S., & Hubeny, I. 1986, BAIC, 37, 129
- Meyer, F., & Meyer-Hofmeister, E. 1982, A&A, 106, 34
- Mihalas, D. 1970, Stellar Atmospheres (San Francisco: W.H. Freeman and Co.)
- Mihalas, D., & Mihalas, B.W. 1984, Foundations of Radiation Hydrodynamics (Oxford: Oxford University Press)
- Mineshige, S., & Wood, J. H. 1990, MNRAS, 247, 43
- Nishiyama, S., Watarai, K., & Fukue, J., 2007, submitted to PASJ
- Peraiah, A. 2002, An Introduction to Radiative Transfer: Methods and applications in astrophysics (Cambridge: Cambridge University Press)
- Ross, R. R., Fabian, A. C., & Mineshige, S. 1992, MNRAS, 258, 189
- Rybicki, G.B., & Lightman, A.P. 1979, Radiative Processes in Astrophysics (New York: John Wiley & Sons)
- Shaviv, G., & Wehrse, R. 1986, A&A, 159, L5
- Shimura, T., & Takahara, F. 1993, ApJ, 440, 610
- Shu, F.H. 1991, The Physics of Astrophysics Vol. 1: Radiation (California: University Science Books)
- Sumitomo, N., Nishiyama, S., Akizuki, C., Watarai, K., & Fukue, J. 2007, submitted to PASJ